UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

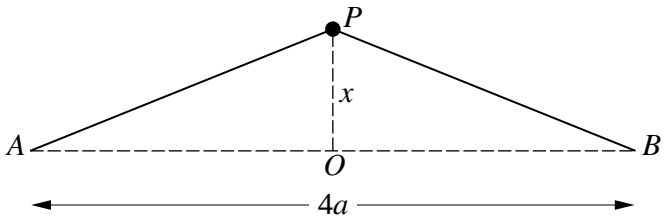
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

1 Two smooth spheres A and B, of equal radii and of masses $3m$ and $6m$ respectively, are at rest on a smooth horizontal surface. Sphere A is projected directly towards B with speed u. The coefficient of restitution between A and B is e. Show that the kinetic energy lost in the collision between A and B is $mu^2(1 - e^2)$. [7]

2



Two light elastic strings, each of natural length a and modulus of elasticity $2mg$, are attached to a particle P of mass m. The strings join the particle to the points A and B which are fixed and at a distance $4a$ apart on a smooth horizontal surface. The particle is at rest at the mid-point O of AB. The particle is now displaced a small distance in a direction perpendicular to AB, on the surface, and released from rest. At time t, the displacement of P from O is x (see diagram). Show that

$$\ddot{x} = -\frac{4gx}{a}\left(1 - \frac{1}{2}\left(1 + \frac{x^2}{4a^2}\right)^{-\frac{1}{2}}\right). \quad [5]$$

Given that $\frac{x}{a}$ is so small that $\left(\frac{x}{a}\right)^2$ and higher powers may be neglected, show that the motion of P is approximately simple harmonic and state the period of the motion. [2]

3 The point O is on the fixed line l. Points A and B on l are such that $OA = 6$ m and $OB = 8$ m, with O between A and B. A particle P oscillates on l in simple harmonic motion with centre O. When P is at A its speed is $4\,\text{m s}^{-1}$, and when P is at B its speed is $3\,\text{m s}^{-1}$. Show that the amplitude of the motion is 10 m and find the period of the motion. [4]

Find the time taken by P to travel directly from A to B, through O. [4]

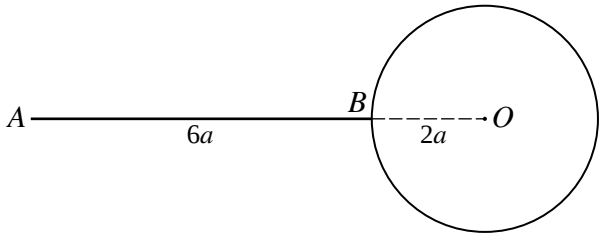
4 A smooth sphere, with centre O and radius a, has its lowest point fixed on a horizontal plane. A particle P of mass m is projected horizontally with speed u from the highest point on the outer surface of the sphere. In the subsequent motion, OP makes an angle θ with the upward vertical through O. Show that, while P remains in contact with the sphere, the magnitude of the reaction of the sphere on P is $mg(3\cos\theta - 2) - \frac{mu^2}{a}$. [4]

The particle loses contact with the surface of the sphere when $\theta = \alpha$. Given that $u = \frac{1}{2}\sqrt{(ga)}$, find

(i) $\cos\alpha$, [2]

(ii) the vertical component of the velocity of P as it strikes the horizontal plane. [5]

5



A uniform rod AB, of mass m and length $6a$, is rigidly attached at B to a point on the circumference of a uniform circular lamina of mass m, radius $2a$ and centre O. The lamina and the rod are in the same vertical plane, and ABO is a straight line (see diagram). Show that the moment of inertia of the system about an axis l through A perpendicular to the plane of the lamina is $78ma^2$. [4]

A particle of mass $2m$ is now attached at B and the system is free to rotate in a vertical plane about the fixed axis l which is horizontal. Initially AB is horizontal, with O moving downwards and the system having angular velocity $\frac{3}{5}\sqrt{\left(\frac{g}{a}\right)}$. At time t, AB makes an angle θ with the downward vertical through A.

(i) Find, in terms of a, g and θ, an expression for $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$. [4]

(ii) Find the angular velocity of the system when B is vertically below A. [3]

6 A random sample of 10 observations of a normal random variable X has mean $\bar{x}$, where

$$\bar{x} = 8.254, \qquad \Sigma(x - \bar{x})^2 = 0.912.$$

Using a 5% significance level, test whether the mean of X is greater than 8.05. [7]

7 The waiting time, T minutes, before a customer is served in a restaurant has distribution function F given by

$$\mathrm{F}(t) = \begin{cases} 1 - \mathrm{e}^{-\lambda t} & t \geqslant 0, \\ 0 & t < 0, \end{cases}$$

where λ is a positive constant. The standard deviation of T is 8. Find

(i) the value of λ, [2]

(ii) the probability that a customer has to wait between 5 and 10 minutes before being served, [2]

(iii) the median value of T. [3]

8 Residents of three towns A, B and C were asked to grade the reliability of their digital television signal as good, satisfactory or poor. A random sample of responses from each town is taken and the numbers in each category are given in the following table.

	Good	Satisfactory	Poor
Town A	24	34	14
Town B	58	60	26
Town C	20	34	30

Test, at the 2.5% significance level, whether grade of reliability is independent of town. [7]

Identify which town makes the greatest contribution to the test statistic and relate your answer to the context of the question. [2]

9 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{2a} & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. Find the distribution function of X. [3]

The random variable Y is defined by $Y = \mathrm{e}^X$. Find the distribution function of Y. [3]

Given that $a = 4$, find the value of k for which $\mathrm{P}(Y \geqslant k) = 0.25$. [3]

10 Engineers are investigating the speed of the internet connection received by households in two towns P and Q. The speeds, in suitable units, in P and Q are denoted by x and y respectively. For a random sample of 50 houses in town P and a random sample of 40 houses in town Q the results are summarised as follows.

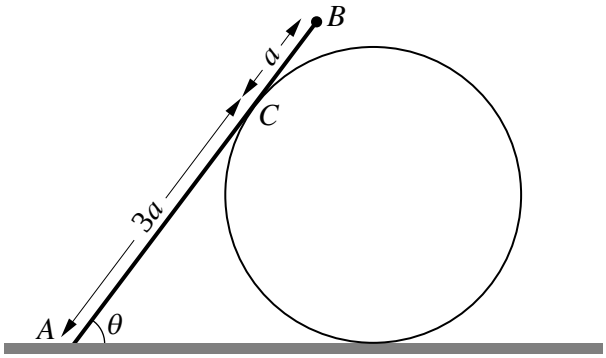
$$\Sigma x = 240 \qquad \Sigma x^2 = 1224 \qquad \Sigma y = 168 \qquad \Sigma y^2 = 754$$

Calculate a 95% confidence interval for $\mu_P - \mu_Q$, where μ_P and μ_Q are the population mean speeds for P and Q. [6]

Test, at the 1% significance level, whether μ_P is greater than μ_Q. [6]

11 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a uniform rod AB, of length $4a$ and weight W, resting in equilibrium with its end A on rough horizontal ground. The rod rests at C on the surface of a smooth cylinder whose axis is horizontal. The cylinder rests on the ground and is fixed to it. The rod is in a vertical plane perpendicular to the axis of the cylinder and is inclined at an angle θ to the horizontal, where $\cos\theta = \frac{3}{5}$. A particle of weight kW is attached to the rod at B. Given that $AC = 3a$, show that the least possible value of the coefficient of friction μ between the rod and the ground is $\dfrac{8(2k+1)}{13k+19}$. [9]

Given that $\mu = \frac{9}{10}$, find the set of values of k for which equilibrium is possible. [3]

OR

For a random sample of 5 pairs of values of x and y, the equations of the regression lines of y on x and x on y are respectively

$$y = -0.5x + 5 \qquad \text{and} \qquad x = -1.2y + 7.6.$$

Find the value of the product moment correlation coefficient for this sample. [3]

Test, at the 5% significance level, whether the population product moment correlation coefficient differs from zero. [4]

The following table shows the sample data.

x	1	2	5	5	p
y	5	3	4	2	q

Find the values of p and q. [5]

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